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Strategies to Improve All Students' Mathematics Learning and Achievement

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Suggested citation:
Arthur, C., Badertscher, E., Goldenberg, P., Moeller, B., McLeod, M., Nikula, J., & Reed, K. (2017). *Strategies to improve all students' mathematics learning and achievement*. Waltham, MA: EDC.

Cover photo:
Burt Granofsky

Graphic Design:
EDC Digital Design Group

Content Editor:
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Adam's World: Reflections on the Achievement Gap

by Courtney Arthur, MEd

Adam ran through the school door and down the hall, late yet again. "Adam," Mrs. Moore, the school clerk, called out, "Come in and get your tardy slip! Don't you know what time school starts? It's the same time every day!" Mrs. Moore sighed and shook her head as Adam took the tardy slip from her. Students had more respect for school in her day.

Grasping the tardy slip tightly, Adam sprinted through the school halls, slowing to a fast walk when he passed open classroom doors. He didn't want to get anyone else mad today! Dodging past his classmates, he could hear them whispering and commenting on his uniform. The stains had been there most of the year and his shoes were without laces. As he raced up the stairs, he realized he hadn't finished his homework. His mom worked the overnight shift last night, so he had stayed at his grandmother's, leaving his belongings back at his house. No breakfast. His stomach growled as he scooted into his classroom.

"Good morning, Adam! Get your stuff put away quickly and bring me that homework," his teacher Ms. Stanley called as 34 of Adam's classmates swarmed around her waving their papers. Adam must have looked at her oddly, as she said again, "Adam? Where's your homework?" Adam tried to explain, but wasn't given much time before Ms. Stanley frowned, slowly shaking her head, and said "Oh, Adam. No homework? You'll need to stay in here and finish your homework with me. No recess for you today." She looked so sad that Adam's heart sank. "What is happening with Adam?" Ms. Stanley thought, but before she could ask him… "Ms. Stanley! Ms. Stanley! Ms. Stanley!" three students ran up to her desk, jockeying for her attention.

As he slowly walked to his seat, Adam's head swirled with the details of the morning. He had only been up for an hour, but that hour contained a rush out the door without his belongings, no food since lunch at school yesterday, witnessing a fight on his way to school, Mrs. Moore getting mad at him, and Ms. Stanley looking sad. His one reprieve, a break from it all—recess—was gone, leaving this day just like a slew of others.

As Adam tried to focus on the math problem and the sheet of other problems that still needed to be completed, his mind kept returning to the fight he had seen. His stomach cramped and his eyes became droopy from a lack of sleep. He sat himself up, straightening in his seat, telling himself to "focus," but the harder he tried, the harder it was. He couldn't shake the awful feeling of all the events leading up to this moment. He was tired, tired of feeling tired, tired of feeling behind and not good enough. After a few minutes, he simply gave up and gently put his head down.

I was Ms. Stanley. I was a math teacher in an urban, low-income district. Every day, I saw huge numbers of students move in and out of the school due to circumstances beyond their control and my control. Losing their home, caregiver living outside of the boundaries, foster care. The list went on and on. Their morning routines were often riddled with stressful events such as lack of food, homework that didn't get done, arguments, no one to kiss them goodbye or even send them off. And often, they were being sent off to a place where they felt isolated, not good enough, and even stupid. Looking back, it was a lonely place for some who were no older than 8 or 9. But this was life; this was normal. Some students were very angry and loud, quick to rise to a fight, and fierce—survival skills that they relied upon to navigate their worlds. Some students, like Adam, were often late, very quiet, and one day they would disappear and never come back to school.

It's easy to talk about Adam when we fret about his low math scores or when we fume about the large gaps in scores between our country's students and those of other nations. But often we don't talk about Adam holistically. It is as if the grinding stress that children in low-income areas face, and its crucial impact on their learning, do not exist. We do talk about making school a welcoming place (which for many, it is not). We do talk about the best strategies for teaching students (let me note that when you are teaching 34 kids, "the best strategies" are often the

Everyone benefits from five minutes of calm.

Offer the class time to free-write or journal their throughts first thing each morning before digging into the curriculum.

Take five minutes for the class to do a little yoga such as child's pose or forward folds next to their desks.

Teach students deep-breathing techniques that they can carry with them all day, each day.

All of these small, calm activities can give students a brief respite and support their emotional well-being.

Acknowledging stress, addressing it, and letting students know it is okay to take a step back and deal with it can be a life-saving tool for some children later down the road.

first thing to go). We even talk about what we should feed students so that they can learn better. But only very recently has our society started talking about how stress impacts students from kindergarten through high school. It is as though stress is something only for adults.

Put yourself in Adam's situation for a moment, as an adult. The constant stress upon stress would eventually eat away at you. There isn't much of you that would want to learn about why the Pythagorean theorem is important, much less cope with the flood of emotions you were feeling in front of 34 peers. This type of stress, the kind that is chronic in nature and develops throughout a young adult's life, can be labeled toxic stress. Toxic stress such as poverty, abuse or neglect, parental substance abuse or mental illness, and exposure to violence has the potential to damage the architecture of the developing brain. It can impair school readiness, academic achievement, and both physical and mental health.

In addition, research has shown that negative emotions like anger reduce achievement partly because they negatively affect higher-order thinking (such as problem-solving, memory, and strategic thinking) and focus attention on a narrow set of behavioral options (Pekrun, Elliot, & Maier, 2009). There is also substantial evidence that cognitive processes are strongly related to achievement; thus, anxiety and anger may disrupt students' ability to recall relevant material (Linnenbrink, 2007; Linnenbrink, Ryan, & Pintrich, 1999). Blair (2002) noted that young children characterized by negative emotionality are likely to have a hard time applying these higher-order processes, simply because their emotional responses do not call for planning or problem-solving. As a result, these skills are underdeveloped and underused. When a young student's experience involving negative emotion leads to focusing on the object of the emotion (such as when they relive a morning event involving conflict) their cognitive resources are diverted away from classroom material, distracting them from learning.

Back when I was teaching, some of my colleagues would say, "They just don't want to learn." In a few very tough moments—dejected, frustrated, and exhausted—I said that myself. My colleagues and I were absolutely wrong. I didn't know how to meet the needs of children like Adam, and far too many teachers still do not know. Not understanding what happens before students run through that front door still puts many teachers at a disadvantage. Lack of district resources, budget cuts, an overwhelming number of students in schools and districts, and limits on the counseling and support offered to students are all complicating factors.

Do you want to work to close the mathematics achievement gap? Here's where to begin: Start paying attention to and addressing the needs of the Whole Student, from preschool all the way up through high school. Change what happens before students walk through the front door, change the support and understanding that they receive once they walk inside the school, and you will change their test scores.

Five Key Characteristics of Effective Diversity Training for Teachers

by Eden Badertscher, PhD

Research shows that diversity training helps teachers improve student achievement and work more effectively with families. Yet teacher preparation programs inconsistently provide diversity training. This spotlights the need to support all current classroom teachers with ongoing, effective diversity training.

From my leadership of EDC's Designing for Equity by Thinking In and About Mathematics (DEBT-M) program, as well as my many years as a mathematics teacher and supervisor, I have found high-quality diversity training to be essential in helping teachers close mathematics opportunity gaps and improve outcomes for students. Unfortunately, high-quality diversity training is not universally available. I agree with researcher Hua-Yu Sebastian Cherng (2016), who stresses the need to re-examine the effectiveness of diversity training on teachers' beliefs and behaviors and challenges us to consider: Are these programs helping teachers meet the needs of students of color and improve outcomes for students?

Over the years, I have identified five key characteristics that are key to effective diversity training for educators. My list is not exhaustive. It reflects my own research and experience in schools. I know that other educators likely have different lists! Presuming the content and format of the training reflects best practice for engaging and supporting adult learners, I would start by asking, Does the program support teachers in the following five ways?

1. Build understanding of assets of other cultures. If you are from a dominant culture, it is easy to note differences between your culture and that of others. Many of these differences are viewed with suspicion, which should come as no surprise given that to examine them in a positive light might threaten the cultural norms you hold dear. Our cultural norms are not all good or often even productive, nor are the norms of non-dominant cultures all bad.

Understanding and honoring the norms of other cultures is one critical pathway to meeting the needs of diverse learners. Stories and the spoken word, the importance of extended family, working together, the use of rhythm and song in daily life, privileging passionate expression, prizing diplomacy…all of these values and norms are, to some extent, challenging to our dominant culture and education system. At the same time, each one has something important to teach us about the children we see in our classrooms every day. If diversity trainings can help us understand and draw on these norms and values, and ultimately sustain them, we can begin to position the talents students bring as assets, and we can make classrooms welcoming and responsive to students of multiple cultures.

2. Challenge "deficit perspectives" of cultures of color. We don't just want to understand and value other perspectives. We must work against the devaluing of other cultures. Unfortunately, prejudice is natural—I hold prejudiced views, you do, we all do. The prejudices that we develop are purely a result of socialization. By default, the dominant culture becomes the "objective" perspective through which members of the dominant culture view other norms. Of course the dominant culture's view is not objective, nor is it better; it happens to be espoused by those currently in power.

It is important for diversity trainings to help us tease apart the aspects of our socialization that are the result of dominant cultural beliefs. For example, a skilled diversity trainer can guide teachers in pinpointing the extent to which student success in mathematics stems from coherence or disconnects between their cultural norms (e.g., communal effort) and school mathematics norms (e.g., individual effort or competition). The most beautiful piece I have ever heard about challenging deficit perspectives is Gloria Ladson-Billings' spoken word piece "Justification," where she highlights why she engages in research with black youth.

3. Demonstrate how the system itself works to maintain difference. As a part of the dominant culture, I don't have to think about my culture. In fact I have had

many colleagues say that we don't really have a "white culture." Almost every person of a non-dominant culture I know would disagree with that statement. Because we as white people are free from thinking about it, we do not see how it influences us, any more than we think about or see the influence of oxygen... until we lose it. But its influence is there and it serves the interests of those in power and undermines the interests of those who are not.

Look at high stakes tests after all. Overall, these tests serve the interests of whites—who are portrayed as intellectually superior—and undermine the interests of many minority group who are portrayed, quite wrongly, as intellectually inferior. Thus tests help maintain the status quo because they privilege the dominant culture. Diversity trainings play a key role in helping us clearly see and question the status quo, they help us see how we all, with or without intent, have a role in maintaining the system because of how the system has shaped us. Acknowledging our own role is an important step in identifying strategies to improve mathematics education for students of color.

4. Recognize that teachers want what is best for their students but need support. Sometimes teachers so acutely want the best for their students that they do things like making the curriculum easier so kids can feel successful. This often happens when teachers have been socialized by the system to see certain children as less capable. I firmly believe that if diversity training is successful in doing #1-3 above, teachers can and will do what is in their students' best interest—which, as the National Council of Teachers of Mathematics notes, includes providing challenging and interesting mathematics for all students, but is also valuing, respecting, and cherishing the students they have and the cultures they represent. Teachers are incredibly caring, yet some of them give up on reaching all students because the system has failed in helping them truly meet students' needs. If we do right by teachers—by providing them with opportunities to learn and the resources and support they need to help all students excel—then they will be prepared to do right by all their students.

5. Focus on learning how to learn in practice. The essential piece to doing right by teachers means empowering them to learn in and from practice, which also means arming them with the tools to challenge the dominant system. There is no silver bullet, but we can help them critically analyze their situation and make purposeful decisions that benefit all of their students. Historically, our schools have provided children with education that is devoid of real problems to solve. Similarly, teacher preparation is often not problem based. As the best educators know, teaching is all about tackling and solving problems through study, experimentation, refinement, re-study, and so on. That means that diversity training, as well as all other forms of professional development, needs to help teachers be learners for their whole careers, teachers who learn from students, who learn from families, who learn from other cultures, and who are willing to try

and at times fail in order to get it right in the long-run. Just like we need to train our students to be learners, we have to teach teachers to be learners in practice. We have to prepare all teachers to be the best problem posers and problem solvers they can be because we have no idea what teachers will face in their classrooms and schools. The best we can do is help them question, understand, value, and keep learning.

There are a variety of programs that do many of these things. Here are some phenomenal places to start as you look for quality programs:

- National Equity Project and their leading and coaching for equity programs
- Pacific Educational Group's Courageous Conversations
- Carnegie Foundation for the Advancement of Teaching's deep focus on Improvement Science and how educators can network and collaborate to learn in and from practice
- Safir & Associates' work at the district and school level

Restorative justice as a practice is transformative for teachers and students. The Algebra Project and The Young People's Project are valuable mathematics-focused programs. Here at EDC we have the Urban Special Education Leadership Collaborative, and I work on an NSF-funded partnership focused on supporting teachers and educators in addressing systemic opportunity gaps in secondary mathematics education.

There are many resources to support schools and districts that are committed to change. Yet the most critical step must be taken internally—recognizing that there is in fact a problem to tackle, coupled with the courage to tackle it.

Ella in Kindergarten: Building on Strengths

by Paul Goldenberg, PhD

Teachers are often encouraged, sometimes compelled, to pre-assess students and then focus on their weaknesses. After all, what's teaching if not helping a child move from weakness to strength? But this story is about a different approach: recognizing and using strengths that are camouflaged by apparent weakness or hidden behind real weakness.

Understanding a child's strengths and helping the child build on those strengths is effective and efficient, but it requires more than check-listing skills or comparing performance to norms. It requires getting to know how the child sees the world. That takes curiosity, focus, and time. And even the most curious, focused, committed teachers—even those with an assistant and a small class—have little time.

Helping a child build on his or her strengths also requires a kind of agenda-free observation, seeing what the child is doing and following the child's lead rather than focusing on what the child isn't doing and leading the child to do it. That, too, is hard, especially in a climate of daily learning objectives, formal documentation of progress, and scales, lists, and rubrics for everything. But Ella's story illustrates the value of watching longer before targeting instruction and remediation. With luck, then, the teaching can build on strengths.

I first met this kindergarten in February. The children were in the hall measuring, in their own footsteps, the length of a whale and some other large sea creatures. Some children, some of the time, carefully paced heel-to-toe, but their zeal to reach the end impelled even the most attentive to speed up, inadvertently spacing out their steps. Their rhythmic counting mostly matched the speedy walking, but sometimes lagged behind or ran ahead.

Impressively, most children were reciting numbers accurately through 70 or so, even if not quite keeping pace with their feet. The teacher asked me to accompany Ella and provide whatever help she needed. Ella wasn't solid at counting. Overall, she seemed to be among the "mathematically weaker" children in this class.

With great focus and meticulous care, Ella placed heel to toe, counting softly up to 29 and then asked me "What comes after 29?"

"Thirty."

She then continued to 39 and asked what came next.

"Forty."

She counted to 49 and said "Fifty?"

"Yes."

She counted to 59, paused, and said "sixty?" When I just smiled, she counted on to 66, the end of her fish.

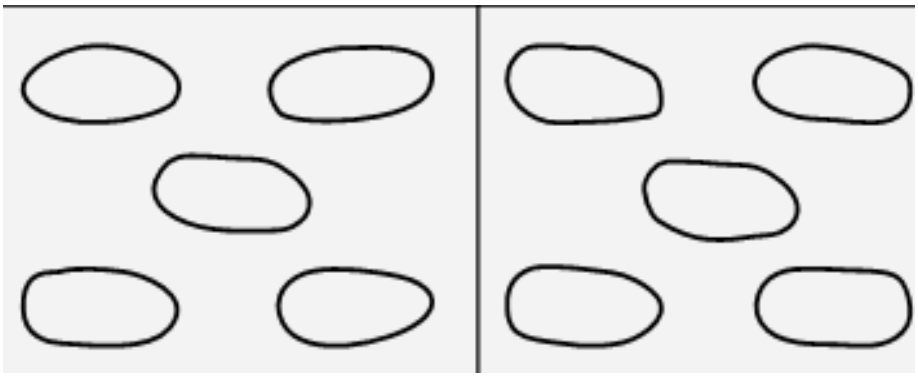
Ella was very self-controlled and organized for a kindergartener (think of that careful stepping). She was also very watchful—her attentive eyes seemed to take in everything—and exceptionally quiet, saying little, but not shy to speak up when she had something important to say or ask like "What comes after 29?"

Though Ella hadn't yet seemed sure of some numbers' names, she soon became very good at adding them. Or perhaps she already was good, and we simply couldn't tell. In March, the class began collecting for charity. Over two months, the children amassed thousands of pennies, a sizeable number of nickels and dimes, many quarters, and even a few dollar bills.

They also amassed a lot of experience with coins. They learned the values of coins, and played with fairly challenging "puzzles," as we called them, like being handed a quarter and being asked to pick out, from a pile of pennies, nickels, and dimes, "that much money" or being asked "Can you make 25¢ with exactly three coins?", "Can you make 25¢ with exactly one coin?", "Can you make 25¢ with exactly five coins?" and so on.

Ella particularly enjoyed these puzzles, and was delighted when I asked her if she could make 25¢ with exactly two coins. Her expression, and her occasional gestures with a dime made it clear that she was clearly thinking, not just hesitating to answer. She continued thinking for seven or eight seconds—quite long if you think about lag times for a kindergartener. And then she gave a confident "no!" with a quizzical smile as if to say "You mean you really can pose problems that can't be solved?!"

Near the end of the year, the children set about counting the money they'd collected. To help organize the counting, the assistant teacher prepared two sheets of paper taped together, with five loops on each sheet. The children could count ten pennies into each loop, count the loops by tens, and dump each hundred produced that way into a separate cup (or, alas, the floor, which received many gifts of pennies) so that they could later count the hundreds.



All the subskills—counting by ones, grouping by 10s, counting by 10s, grouping by hundreds, and counting by hundreds—had been developed over the course of the year in various ways including their "calendar time" activities.

Some children initially just counted coins, regardless of value. But, with an example or two, all understood that a loop on their sheet could contain one dime or two nickels or anything that made exactly 10¢. There were also a couple of children who didn't at first understand the goal and, after counting 100 pennies onto the sheet, dumped the 100 into an existing set of 100 rather than create a new one or, worse, dumped them back into the money-jar as if counting to 100 was the only goal.

Ella had no such confusions. She filled each loop with exactly 10¢, easily choosing whatever coins she needed.

When she encountered her first quarter, she looked at me as if to ask what to do. I said nothing but my slight shrug was as if to say "you'll figure something out." That was just enough permission for her to continue what she was reaching out to do.

With a big smile, Ella said, "I'll just get another quarter and that'll be this whole page." She moved beyond the sub-structure of the page—the loops that must contain only 10¢—and saw the structure of the entire page, the five tens that make the page worth 50¢.

This reminds me of a boy who had great interest in books well before school, appeared to read silently with understanding, but, in the early grades, was halting and faulty at out-loud reading. Because reading ability was gauged by out-loud reading, he and his teachers all assumed that he was "no good at reading."

The problem was the medium by which they gauged reading ability: out-loud reading was not giving valid information. With this boy, as with Ella, we can't retroactively know what skills were already there before we first saw them. However, the speed with which full fluency was achieved when these children bloomed strongly suggests that more was already present than we had been able to see.

Ella might have already seen in her head, perhaps as a visual image, a structure of addition without knowing all the number names that would allow her to say what she "saw." When she saw a pair of hands with a display of fingers, she fluently knew how many fingers were "hidden" to make that display.

She also seemed to have a sense of how much "overflow" there would be if fingers were added to that display, pushing the total past ten. But she could not give any evidence of knowledge of larger numbers until she built a structure for their names, knowing what came after 29, 39, 49, and connecting those with the litany of counting by tens that she already knew as a separate skill.

When Ella had that, she could then, pretty suddenly, show astonishing fluency with numbers into the multiple hundreds. The suddenness, like the suddenness of the boy's acquisition of exceptional reading fluency in fifth grade, is what makes it seem as though she had the essential underlying abilities and understandings all along.

It doesn't really matter whether the understanding was already there and unseen or whether it just grew quickly: the point is don't assume that what you see is all there is. Skills do not develop at the same time or even necessarily in the same order in all children who eventually reach total competence.

Children who are "late starters" sometimes catch up almost instantly once they do start. We sometimes call these kids "slow learners" but the learning isn't slow. It may require removing a roadblock. It likely requires working through strengths—Ella's fascination and facility with the cognitively challenging coin puzzles—rather than limiting a child's input by focusing on weaknesses.

My own personal story is another illustration. My 7th and 8th grade math experience was, to my memory, filled with calculating compound interest, for which my interest was zero and my competence worse. I labored with multiplication facts and found the manual effort of writing difficult, messy, and unreliable. As an apparent math-dud, I was placed in the lowest 9th grade algebra class.

But I read word problems fluently unlike my classmates. The school, for its sake, wanted me out of that class. They couldn't fit me in the middle level—it was already full—so they ditched me in the top class. Where 7 x 8 and 9 x 6 were muddled in my head and $673.98 compounded twice at 27% had been torture, $2x - 5 = 30 - 3x$ was obvious—the numbers were easy and the logic was beautiful. Being given fancier mathematics was the "remediation" I needed.

From then on I excelled in mathematics through calculus in high school, and attended all sorts of special institutes in mathematics for "gifted" high schoolers (including an abstract algebra course, and what was, way back then, computer science). But my "gift" in mathematics wasn't apparent while we were just calculating compound interest! High school algebra became a foundation from which I learned even the arithmetic. I was lucky. I could easily have been left reviewing "basics" and never have seen serious mathematics.

Even while Ella was still looking inept at the subject, her teachers noticed two valuable attributes that she brought to her mathematics. Ella observed so closely, one almost felt studied by her. All that observation was bringing in data; we just couldn't yet tell what she knew. And, although Ella could be quite independent and was not as garrulous as many kindergarteners are, she asked when she needed information. This kind of self-advocacy, too, was promising.

If one judged Ella's mathematical aptitude by her performance on skills such as counting, then even by the second half of that year when I first met her, it looked poor. For her, it was especially important that her teachers understood that skills don't develop in a fixed order. Otherwise, they might not have posed the challenging puzzles that Ella found so intriguing and was so exceptional at solving—the fancier mathematics that was the effective "remediation" for Ella and that showed what Ella could do.

Math for All: High-Quality Mathematics Instruction for Students with Disabilities

by Babette Moeller, PhD and Matthew McLeod, MEd

Standards-based reform holds great promise for increasing the rigor and quality of mathematics education for students with disabilities. For nearly two decades now, national content standards for mathematics have emphasized the need to make high-quality mathematics instruction accessible to students with disabilities. For instance, the Common Core State Standards in Mathematics clearly recognize that all students, including those with disabilities, "must have the opportunity to learn and meet the same high standards if they are to access the knowledge and skills necessary in their post-school lives" (Common Core State Standards Initiative, 2010).

Reflected in the national standards is the recognition that all students, including those with disabilities, can learn mathematics, and that students may vary in how

they learn best. For students to be successful in mathematics, teachers need to offer them appropriate opportunities to learn. Offering identical instruction to every student will no longer be sufficient for teachers to help every student attain standards-based learning outcomes. To date, however, the promise of standards-based reform has not been readily fulfilled.

On the most recent mathematics assessment conducted by the National Assessment for Educational Progress (NAEP), almost half of all 4th graders with a disability (45 percent) and over two-thirds of all 8th graders with a disability (68 percent) scored below the basic achievement level (U.S. Department of Education, 2015). By way of comparison, only 14 percent of the 4th graders and 23 percent of the 8th graders without disabilities scored below the basic achievement level. As a result, the opportunities for these students to excel in an increasingly technology-based society and to pursue careers in science, technology, engineering, and mathematics have been profoundly limited; both the scientific and humanities enterprises have been denied their talents and contributions.

Following the Individuals with Disabilities Education Act (1997), we define disability broadly, to include mental retardation, hearing impairments (including deafness), speech or language impairments, visual impairments (including blindness), serious emotional disturbance, orthopedic impairments, autism, traumatic brain injury, other health impairments, or specific learning disabilities and developmental delays.

Research shows that teacher quality is the single most powerful influence on student learning (e.g., Darling-Hammond & McLaughlin, 1995; Nye, Konstantopoulos, & Hedges, 2004; Rivkin, Hanushek, & Kain, 2005). Yet teachers often are not well-prepared to implement standards-based mathematics education with the heterogeneous groups of students that are being served in general education classrooms, including students with disabilities and students with different capabilities, needs, and learning styles.

According to a recent national survey of science and mathematics teachers, less than half of the elementary school math teachers (42%) felt well prepared to plan instruction so that students at different levels of achievement can increase their understanding of the ideas targeted in each activity; less than a quarter of elementary school math teachers reported feeling well-prepared to teach students with learning (23%) or physical (16%) disabilities (Banilower, Smith, Weiss, Malzahn, Campbell, & Weis, 2013). Moreover, only a third of the teachers (33%) reported receiving professional development (PD) in the past three years on how to provide alternative mathematics learning experiences for students with special needs. To achieve the full promise of the national standards, targeted, well-designed PD efforts are required to better prepare teachers for teaching high-quality mathematics to the full range of students they will encounter in general education classrooms.

In response to the need to improve teacher preparation to teach high-quality mathematics to the wide range of students in their classrooms, EDC staff developers and researchers have developed two sets of intensive PD programs that are intended as resources for teacher leaders. Math for All (MFA) for grades K-5, and Addressing Accessibility in Mathematics (AAM) for grades 6-8, are designed to help teachers provide all students, including those with disabilities, with access to significant mathematics content.

While serving different audiences (elementary versus middle-school teachers) and covering different mathematics topics, both programs utilize several common approaches to professional learning, including:

- Fostering collaboration between general and special education teachers
- Engaging teachers in planning, and reflecting on, mathematics classroom practice
- Helping teachers better understand individual students' strengths and needs in mathematics

These approaches are well aligned with standards for professional learning (Learning Forward, 2011), research on effective professional development (e.g., Darling-Hammond, Hyler, & Gardner, 2017), and the recently enacted *Every Student Succeeds Act* (ESSA, 2015) which strongly emphasizes high-quality professional development (PD), that is "sustained (not stand-alone, 1-day, or short-term workshops), intensive, collaborative, job-embedded, data-driven, and classroom-focused ..." (S.1177, §8002 [42]).

About 13% of public school children between the ages of 3 and 21 have disabilities (National Center for Education Statistics, 2017).

The majority of these students spend 80% or more of their time in general education classrooms (Jones, Buzick, & Turkan, 2013), and 95% of general education teachers currently teach students with disabilities, or have done so in the past (Pugach, 2006).

As shown in this video about Math for All, we have found that bringing general and special education teachers together for shared professional learning experiences can be a powerful approach for improving the accessibility of mathematics instruction for students with and without disabilities. First, general and special education teachers do not always use the same math curriculum even if they teach at the same grade level. As a result, what a special education student learns in a resource room can be quite disconnected from what his or her peers are learning in the general education classroom. Bringing general and special education teachers together for shared professional learning and collaborative lesson planning can help facilitate better alignment between mathematics instruction that happens across different settings (resource room, general education classroom, special education classroom) within a school.

Second, general and special education teachers bring important and complementary areas of expertise that are both important for planning mathematics lessons that are rigorous and that provide multiple entry points that are attuned to individual students' strengths and needs. Special education teachers have expertise in disability and individualizing learning experiences, and general education teachers tend to be more familiar with current issues in mathematics teaching and learning. Moreover, given their different areas of expertise, general and special education teachers often perceive students through different lenses, and sharing their perspectives of individual students can help these teachers gain a more well-rounded understanding of the learning profiles of these students.

Third, we have found that fostering teacher collaboration can help enhance the sustainability of PD efforts, as working in partnerships with colleagues provides a

mechanism for teachers to continue their professional learning while holding each other accountable for participation. Last but not least, teacher collaboration can serve as a powerful model for student learning, as illustrated by the following quote from a teacher who participated in the Math for All PD:

What I enjoy most about Math for All is working with a team of other thoughtful teachers in terms of planning and anticipating challenges and then working together to be problem-solvers. I feel like that dynamic of working with other adults and becoming problem-solvers is a good model, and it's practice for what we want our students to be in the classroom. So if we want our students to become problem-solvers, we need to work together, work collaboratively, and share ideas with each other. We are practicing that as adults, and I feel like we don't have as many opportunities as we like to engage in that kind of work, and Math for All has brought us together to do that. **Grade 5 Math Teacher, Chicago Public Schools**

As powerful as the collaboration between general and special education teachers can be, it is not a way of working that will come naturally for all teachers. As teachers often spend many hours working in isolation, they may need to learn how to collaborate with others. There are a number of strategies that we use in the Math for All PD to facilitate teachers' work with each other, including:

- Establishing a community of trust through ice-breaker activities and norms for working with each other
- Providing teachers with a common framework and language for understanding students' mathematics learning and development
- Introducing teachers to a process for collaborative lesson planning that scaffolds their interactions and work with each other

The success of the collaboration between general and special education teachers is also dependent on systems factors, such as the availability of common lesson planning time, instructional leadership, and a collaborative school culture. Working with school leaders to help nurture the conditions that are conducive to teacher collaboration clearly is an important part of making the collaboration between general and special education teachers a success.

Supporting English Learners in the Mathematics Classroom

by Johannah Nikula, EdM

When I describe my work to support mathematics teachers with students who are English learners (ELs), I often hear a similar reaction. People respond that mathematics class seems like a natural place for students who are ELs to thrive. Mathematics is about numbers and pictures (of shapes), not words. Why would mathematics teachers need support?

The sentiment that mathematics class is a natural and important place for EL students to excel rings true. Yet words and language really are an integral part of students' experiences in learning mathematics. In fact, mathematics lessons can be packed with both academic and everyday language. For example, students encounter word problems and task instructions. And teachers and students alike draw upon words to explain their mathematical thinking. For this reason, mathematics lessons are a fruitful place to carefully intertwine support for both the students' language development and their mathematical learning. How can mathematics teachers make the most of this rich learning environment for students who are ELs?

As with all good teaching, teachers' knowledge and curiosity about the strengths and needs of each student are essential to providing students with appropriate support. The first thing that mathematics teachers can do is be aware that students who are ELs come with a wide array of experiences and needs, so that they can capitalize on those experiences and provide appropriate supports related to the needs.

Language Proficiency: For example, teachers should find out students' English-language proficiency level as defined by their state/district. WIDA (World-Class Instructional Design and Assessment) levels provide one example. Teachers must keep in mind, however, that these levels do not describe all the strengths and needs of individual students. Students will speak different languages, which, for example, may or may not include cognates that could be helpful in the mathematics classroom (e.g., cuadrilátero in Spanish). Furthermore, teachers must work to understand the nuances of individual students' language use in the mathematics classroom. Students may vary in their experience with writing and speaking (forms of productive language) and reading and listening (forms of receptive language). To ensure that each student fully participates in the classroom mathematical discourse, it is essential that teachers fully integrate both productive and receptive language into mathematics lessons.

Educational Background: In addition to students' language proficiency, teachers should seek to understand students' previous school experiences. Some students may have had interrupted formal schooling. Others may have been in the school system for years, even though their English language skills are not yet proficient. The kinds of support that these two types of students need will be quite different.

By getting to know each student's needs, teachers can integrate strategies that will enable EL students to engage deeply in mathematics learning. For example, in Ms. Henderson's class, Thuy benefits from Ms. Henderson's use of pictures or acting out the "plot" of a word problem. Alejandro does not need these supports, but he does need help explaining his mathematical thinking. Ms. Henderson gives Alejandro opportunities to rehearse his explanations with a partner and provides him with sentence starters that scaffold the wording of the explanations.

In particular, three instructional principles that can guide thinking about support for EL students have emerged from our work:

1. Engage students with challenging mathematics tasks
2. Support the development of students' language in mathematics
3. Employ and support multimodal representation or communication (speaking, writing, diagramming, gesturing, etc.)

Aspects of these three principles can and will benefit all students, but they are essential for EL students. Engaging all students, including those who are ELs, in challenging mathematical work (Principle 1) means providing sufficient language

access to students so that their mathematical learning opportunities are supported rather than compromised. The language may need to be simplified, but the mathematics should not be.

Supporting EL students' language development *(Principle 2)* is accomplished through the careful use of instructional strategies such as word banks that are jointly constructed by students and teacher within the context of work on a particular mathematics task: sentence starters, revoicing, multiple reads of mathematical problems, etc. Students also develop their mathematical language through opportunities to produce that language, which means they must be given frequent opportunities to use written language and hear oral language individually, in pairs, and in larger group discussions.

The use of multimodal representation in the mathematics classroom *(Principle 3)* supports both of the first two principles. For example, opportunities to explain thinking around a visual representation, such as a mathematical diagram, permit the student to use the diagram to understand the mathematics and also provide visual support for the student's explanations of his or her thinking.

Enacting these principles and the associated instructional strategies can support EL students and encourage them to be participants in the mathematical work and discussion in their classrooms. As we employ strategies to support EL students' access to this participation, each idea that a student verbalizes or expresses through a drawing or notation represents a seed of potential for further mathematical thinking and communication by that student. In a task about sharing candies among children, for example, a student may say aloud "1/3 of candies" or draw a tape diagram that shows how each child's candies relate to the others. Even if a full explanation is not provided in one of these cases, it provides the opportunity for follow-up questions from the teacher or from other students about what quantities from the original task are represented and what can be learned from these quantities.

It sometimes takes work to follow the line of thinking that a student is trying to express, especially if the student struggles with verbalizing his thinking. Even after determining what the student is thinking, it can be tricky to figure out which instructional strategies will keep the student engaged in challenging mathematical tasks while developing their language (Principles 1 and 2). That work is well worth the effort, however. It supports the teacher in determining the additional follow-up and support needed to build on the student's thinking. And best of all, it supports the student in continuing to develop her own mathematical thinking and communication. I've found that it's incredibly helpful to keep your focus on the mathematical ideas, or seeds of thinking, that are embedded in the topic that students are thinking and talking about. When you do that, you can work to employ the three principles described above—keeping students engaged in challenging mathematical work, while developing their communication skills with the aid of a variety of modes of communication and visuals to support their learning.

Helping Children from Low-Income Communities Become Young Mathematicians

by Kristen Reed, MEd

A strong foundation of early mathematics knowledge is key to the school readiness and success of all young children, and all children are budding mathematicians. For example, before children are able to count or have the language to compare quantities, they notice which pile of cookies has more and which has less. They are aware of the phenomena of "more or less," they are interested in it, and they are ready to dive into the early language and math experiences that foster their later school success. Their teachers need to be ready, too; although young children have the capacity and interest to learn meaningful mathematics, they need adult support.

For young children from low-income communities, knowledgeable teachers and high-quality preschool mathematics experiences play an especially critical role in

successful math learning and achievement later on in school (Geary, 2013; Geary, Hoard, Nugent, & Bailey, 2013; National Mathematics Advisory Panel, 2008). I agree with Alyse Hachey's observation that, "Early childhood mathematics education (ECME) represents a powerful and currently underutilized weapon in the war on poverty." Unfortunately, many early childhood teachers do not have the training and support they need to provide children from low-income communities with the high-quality math learning experiences they need to thrive throughout school.

In our Young Mathematicians program and study at EDC, Jessica Young and I have been identifying and testing new strategies to help Head Start teachers who work with children in low-income communities integrate challenging, high-quality mathematics learning into their classrooms in a fun way—through playing games. Our professional development for teachers helps them use a set of carefully designed, developmentally appropriate, and mathematically challenging games to support young children's early mathematics learning (Reed & Young, 2017). The professional development also has a special focus on building teachers' capacity to support children's "mastery motivation"—the ability to persist at challenging tasks—a skill that is key to school readiness (Young & Reed, 2017). You can take a quick look at how the games work, and how they impact children's learning, in this great video produced by our team and EDC videographer Burt Granofsky: "Teaching Persistence."

The games are both simple to use and very engaging. Some are quick and use everyday materials; others use a game board and require more extended play. All of the games share the following characteristics:

- They can be repeated many times, and their difficulty can be decreased or increased to sustain challenge as children gain proficiency.
- They engage children in problem solving, puzzling, and discussing strategies as they play.
- They collectively focus on counting, operations, algebraic thinking, and geometry.
- They promote children's persistence.

In one of the games, Jumping on the Lily Pads, children learn to identify numerals, practice counting sequences forwards and backwards, hop space-by-space on a game board, and "count on." Researchers have found that prompting students to count on from a given number rather than to just count-from-1 assists with learning the names of the numbers and in understanding their magnitude (Laski & Siegler 2014; Ramani & Siegler, 2008; Siegler & Ramani, 2008; Siegler & Ramani, 2009; Whyte & Bull, 2008).

The child tosses a dot cube (with only 1 and 2 to start) and moves a frog game piece that number of "jumps" along the lily pads, which have the numbers 0 to 10 written

on them. Then the teacher, caregiver, or a friend jumps his or her frog. Children can play this game simply to start, and then expand it by jumping backwards on the numbers to return "home" and expanding the board to include numbers 11 to 20. In some classrooms, they have adapted the game to fit what they have been studying—for example, changing it to a bear hopping on bear paw prints. In other classrooms, teachers have made life-size boards so children can be the ones hopping.

We are seeing some promising results from the use of these games, especially when used regularly. In addition to promoting persistence and problem-solving and fostering early math skills and knowledge, the games help expand children's vocabulary and foster key social and emotional skills. For example, games help children learn how to be patient, take turns, work well with others, and succeed and struggle graciously and respectfully. All of these ways of interacting with others are important not just in preschool or in math class, but in life.

You can explore the games on our Young Mathematicians website, and you can also learn more about our ongoing research and professional development work on the site. Right now, with the support of the Heising-Simons Foundation, we have expanded our work to include helping teachers support families in promoting their children's math learning at home and studying the role of teacher-child interactions and teacher talk in successful early math education. As we move forward, we will be looking for more ways to help teachers close mathematics education opportunity gaps for children by designing and testing games and activities that don't just support their learning, but influence their belief about learning and their own intelligence. With the help of their teachers, all young children—regardless of their families' incomes—can become very strong mathematicians. When they do, it will be a big win for all of us.

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